UNDEREMPLOYMENT EQUILIBRIUM IN INTERNATIONAL TRADE*

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The current interest in variations of investment and consumption, while affecting almost all branches of economic analysis, has wrought more far-reaching changes in the theory of closed economies than in the analysis of international equilibrium. Discussions of trade between countries continue to emphasize movements of prices, interest rates, and the rate of exchange, whereas the modern analysis of national income relegates money prices, money wages, and the like, to a secondary position.[1]

Good reasons for the difference in approach no doubt exist. The balance of payments, the level of commodity exports and imports, short-term capital movements, and similar factors, may respond much more readily to changes in *relative* prices and interest rates than the variables of a closed economy respond to changes in *absolute* prices, money wages, and interest rates.

But frequently it becomes important to know whether there are equilibrating forces in international trade other than price adjust-

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[1] One may distinguish three broad groups of trade theories: (1) Those which rely mainly upon price-level, interest-rate, and exchange-rate adjustments. (2) Those which emphasize the influence of shifts in monetary purchasing power upon the distribution of resources, the balance of trade, and so on. (3) Those which consider the dependence of the level of real income upon the international situation. Examples of the first group may be found in the works of the early English economists, although Professor Viner has shown that the "classical" economists were not unmindful of purchasing power effects (J. Viner, *Studies in the Theory of International Trade*, pp. 290–311). Shifts of purchasing power receive much attention in Professor Bertil Ohlin's *Interregional and International Trade* (cf. especially, Chapters IV, XVIII, XIX, and XX.) The work of Mr. R. F. Harrod provides perhaps the best example of the real-income approach (*International Economics*, Chapter VI and VII; *The Trade Cycle*, pp. 145–158). My analysis is confined entirely to the third approach.

ments,[2] and if so, how these other forces operate. Thus, for example, if two countries which are trading with each other have substantial amounts of unemployment, and if, because of minimum-wage laws, union restrictions, or custom, money wages are relatively inflexible, equilibrium of the balance of payments and the balance of trade cannot be obtained readily by adjustments of prices and money costs. If it is assumed, in addition, that the two countries are on gold standards with sufficient reserves so that central-bank policies are not affected by gold movements, then changes in the rate of interest and the exchange rate may be ruled out. The consistency of domestic changes with the maintenance of external equilibrium will then depend upon the way in which variations of investment and consumption in one country react upon investment and consumption in others. The effects of such changes are isolated, in the following analysis, by setting up a model of trade between two countries in which variations of prices, interest rates, and the rate of exchange are impossible.

Specifically, price and wage-rate changes are eliminated by assuming the existence of unemployed resources in each of the trading countries, with perfect competition, no economies or diseconomies of large-scale operation, and a perfectly elastic supply of the unemployed resources. I assume also that the central banks offer to borrow and lend at fixed interest rates (not necessarily the same in each country) so that no changes in either economy can react upon capital costs. Finally, it is possible to abstract from exchange-rate fluctuations by assuming that the central banks agree to buy and sell foreign balances at a fixed rate, replenishing such balances when necessary either by interbank loans or by gold movements. Alternatively, rates of interest might be introduced with the assumption that the central banks adjust differences in rates so as to equate the amount of foreign lending to the balance of trade. Such an assumption complicates the problem, however, and obscures the investment-consumption relations which I wish to analyze. I shall therefore confine myself to the simpler model.

It is immaterial what unit of measurement of income, investment, and consumption is adopted, since prices, monetary costs, and effort per unit of output are constant in both countries. Suppose that the wage unit in one of the two countries is used as a unit of measure in both. The two economies may be designated Y and Y'; in what follows all letters with a prime ($'$) affixed refer to functional relations in country Y', and all letters without the prime refer to country Y.

Income, as usual, is defined as the sum of consumption and invest-

[2] The term "price adjustment" denotes, in the present context, any movement of monetary variables and is understood to include interest- and exchange-rate fluctuations.

ment. The consumption function for each country is divided between consumption of domestic goods and consumption of foreign goods, both depending upon income. Thus for country Y we may write $u=u_1(y)+u_2(y)$ where y represents total income, $u_1(y)$ consumption of domestic goods, $u_2(y)$ consumption of foreign goods, and u total consumption. Likewise, investment is divided between net increases in producers' goods and stocks by domestic manufacturers and increases in foreign claims arising out of favorable trade balances, as follows: $v=v(y)+u_2'(y')-u_2(y)$. In this last equation v denotes total investment, $v(y)$ domestic investment (assumed to depend upon the level of national income),[3] and $u_2'(y')-u_2(y)$ foreign investment (or disinvestment) arising out of a favorable (or unfavorable) trade balance;[4] $u_2'(y')$, of course, is the consumption by country Y' of goods produced in Y.

The complete system of equations for the two countries may be obtained by writing down a set for Y' similar to that for Y.

$$
\begin{aligned}
u &= u_1(y) + u_2(y) + \beta + \mu, \\
v &= v(y) + u_2'(y') - u_2(y) + \alpha - \gamma - \mu, \\
u' &= u_1'(y') + u_2'(y'), \\
v' &= v'(y') + u_2(y) - u_2'(y') + \gamma + \mu.
\end{aligned} \tag{1}
$$

Since $y \equiv u+v$ and $y' \equiv u'+v'$, equations (1) represent four equations in the four unknowns u, v, u', and v'. If certain stability conditions to be discussed below are satisfied, this fundamental system determines the equilibrium of investment and consumption in each country, and hence total incomes and the balance of trade. The parameters α, β, γ, and μ are inserted to represent changes which will be analyzed later.

Equations (1) represent a *static* scheme in the sense that the variables are not dated. Suppose the equilibrium values of this system are u_o, v_o, y_o, u_o', v_o', y_o'. If time sequences are assigned to the variables of (1) we no longer have a set of simultaneous equations with equilibrium values, but a set of difference equations defining time paths for all the variables of the system. Suppose that initially the variables have some

[3] It is important not to confuse this income-investment relation with the acceleration principle. I follow Lange, here, in assuming that, in the short run, entrepreneurs' expectations, and hence rates of private investment, vary directly with changes of income. Such an interpretation is not inconsistent with the fact that private net investment over a longer period is independent of the level of economic activity.

[4] A country may import, of course, for purposes of investment as well as for consumption. I have excluded this assumption from my model because the complexity occasioned by its addition is accompanied by no change in matters of principle.

values other than those obtained by the solution of equations (1). I shall follow Frisch[5] and Samuelson[6] in defining the system as stable if, for initial values of the variables slightly different from the solutions of the static equations, the time paths tend to approach the equilibrium values $u_o, v_o, y_o, u_o', v_o', y_o'$. Thus stability depends not only upon the character of the static equations (1), but also upon the nature of the assumed dynamic system.

The dynamic system to be used henceforth is derived from the following assumptions: (1) Consumption of both foreign and domestic goods in period t depends upon income of the period $t-1$. (2) Entrepreneurs' expectations of future profits, and hence the rates of present domestic investment, depend upon income of the previous period. The difference equations corresponding to the static system (1) are then given by (2):[7,8]

$$
\begin{aligned}
u(t) &= u_1[y(t-1)] + u_2[y(t-1)], \\
v(t) &= v[y(t-1)] + u_2'[y'(t-1)] - u_2[y(t-1)], \\
u'(t) &= u_1'[y'(t-1)] + u_2'[y'(t-1)], \\
v'(t) &= v'[y'(t-1)] + u_2[y(t-1)] - u_2'[y'(t-1)].
\end{aligned} \tag{2}
$$

Since income of period t is equal to consumption plus investment of period t, we may also write:

$$
\begin{aligned}
y(t) &= u_1[y(t-1)] + v[y(t-1)] + u_2'[y'(t-1)], \\
y'(t) &= u_1'[y'(t-1)] + v'[y'(t-1)] + u_2[y(t-1)]
\end{aligned} \tag{3}
$$

Explicit solutions of (3) for y and y' as functions of time should enable one to state the conditions which our consumption and investment

[5] Frisch, Ragnar, "On the Notion of Equilibrium and Disequilibrium," *Review of Economic Studies*, Vol. 3, No. 2, February, 1936, p. 102.

[6] Samuelson, P. A., *The Foundations of Analytical Economics* (unpublished), Chapter VIII.

[7] The results obtained in the remainder of this paper may seem to be valid only for the assumptions of equations (2). I have experimented, however, with two other systems. In the first of these a lag of one period in the expenditure of income behind its receipt is postulated, with investment in a given period dependent upon income of the same period. Income, in the second alternative system, is consumed (or saved) in the period in which it is received, while changes of investment lag one period behind changes of income. For the normal case in which the marginal aggregate propensity to consume and the marginal propensity to invest are both less than unity, the two alternative systems place substantially the same restrictions upon the character of the static scheme as equations (2). The final conclusions therefore possess greater generality than the method of analysis suggests.

[8] Parameters are omitted from (2) because they do not alter the conditions of stability.

functions must fulfill for stability of the world economy. A general solution is impossible, however, since the exact forms of $u(y)$, $u'(y')$, etc., are not known and even if known would probably be nonlinear. But if displacements from equilibrium are small, the functions of (3) may be expanded about y_o and y_o' and all except first-order terms neglected thus:[9]

$$
\begin{aligned}
y(t) - y_o &= (u_{1y} + v_y)[y(t-1) - y_o] \\
&\quad + u'_{2y'}[y'(t-1) - y_o'] + \cdots, \\
y'(t) - y_o' &= (u'_{1y'} + v'_{y'})[y'(t-1) - y_o'] \\
&\quad + u_{2y}[y(t-1) - y_o] + \cdots,
\end{aligned}
\tag{4}
$$

where the letter subscripts indicate differentiation evaluated at the equilibrium points y_o and y_o'. Equations (4) are linear difference equations with constant coefficients whose solutions may readily be obtained. Before discussing the conditions of convergence, however, I wish to consider two other cases with stability conditions somewhat more simple than those of (4).

Suppose, first, that the economy Y is in complete isolation and let $u(y) \equiv u_1(y) + u_2(y)$ be the aggregate consumption function, now relating exclusively to domestic goods. The linear difference equation for this simple case is $y(t) - y_o = (u_y + v_y)[y(t-1) - y_o] + \cdots$. Its solution is $y(t) = y_o + [y(0) - y_o](u_y + v_y)^t$, where $y(0)$ is the initial value of income at time $t = 0$. In order that $y(t)$ shall converge to the value y_o it is necessary and sufficient that $u_y + v_y < 1$. In words, for a single economy to be in equilibrium in isolation, the marginal aggregate propensity to consume plus the marginal propensity to invest must be less than unity.

Next suppose that the economy Y is trading with the rest of the world, but that it is such a small part of the world economy that reactions of other countries to changes in Y's demand for foreign goods may be neglected. The exports of Y to the rest of the world, $u_2'(y')$, may then be taken as data. Again, the linear difference equation corresponding to (4) is $y(t) - y_o = (u_{1y} + v_y)[y(t-1) - y_o]$. The solution of this equation converges to the value y_o provided $u_{1y} + v_y < 1$. Thus in order that an economy trading with the rest of the world be stable under conditions in which world reactions may be neglected, it is necessary that the marginal propensity to consume domestic goods plus the marginal propensity to invest be less than unity. The size of the marginal propensity to consume foreign goods and hence of the marginal aggregate propensity to consume is immaterial. *Ceteris paribus*, changes in the consumption of foreign-made goods will not affect income in Y,

[9] Cf. Samuelson, P. A., *op. cit.*, pp. 200–226.

as may be seen from (3). Consequently, a very large marginal aggregate propensity to consume is compatible with perfect stability provided only that the marginal propensity to consume domestic goods and the marginal propensity to invest domestically be sufficiently small.

In the usual case, however, the assumption that other things remain equal is not justified, since greater imports (in Y) will change the incomes of other countries and their demands for Y goods. Thus the more general case of trade between two countries with all reactions taken into account must be considered. The time paths of income in Y and Y', for small deviations from equilibrium, are given, as previously noted, by equations (4), the solutions of which are

$$\begin{aligned} y(t) &= y_o + A\rho_1{}^t + B\rho_2{}^t, \\ y'(t) &= y_o' + C\rho_1{}^t + D\rho_2{}^t, \end{aligned} \tag{5}$$

where ρ_1 and ρ_2 are roots of the quadratic equation

$$\begin{vmatrix} (u_{1y} - v_y) - \rho & u'_{2y'} \\ u_{2y} & (u'_{1y'} + v'_{y'}) - \rho \end{vmatrix} = 0, \tag{6}$$

and where A, B, C, D are constants dependent upon the initial values of y and y'. It is clear from (5) that $y(t)$ will approach y_o as t becomes large provided ρ_1 and ρ_2, the roots of (6), are less than unity. Necessary and sufficient conditions for this to be true are

$$u_{1y}+v_y+u'_{1y'}+v'_{y'}<1+(u_{1y}+v_y)(u'_{1y'}+v'_{y'})-u_{2y}u'_{2y'}<2. \tag{7}$$

The inequality $u_{1y}+v_y+u'_{1y'}+v'_{y'}<2$ shows that at least one of the two countries must be in stable equilibrium without considering reactions of the other; i.e., either $u_{1y}+v_y$ or $u'_{1y'}+v'_{y'}$ must be less than unity. Suppose this stable country is Y'. The first of the inequalities of (7) may then be written

$$u_{1y} + v_y + \left(\frac{u'_{2y'}}{1 - u'_{1y'} - v'_{y'}}\right)u_{2y} < 1, \tag{8}$$

which shows that the other country must also be stable with reactions of the first ignored; i.e., $u_{1y}+v_y$ must also be less than unity. Thus stability of the world economy with all reactions considered implies stability of each economy with the reactions of the other ignored. Other implications of our general stability conditions are as follows:[10] (1) If both countries are stable when isolated the world economy will likewise be stable; i.e., (7) will always be satisfied if $u'_{1y'}+u'_{2y'}+v'_{y'}$ and

[10] The proof is so simple in each case that I have left it to the reader.

$u_{1y}+u_{2y}+v_y$ are both less than unity. (2) If both countries are *unstable* when isolated the world economy will likewise be unstable; i.e., (7) will never be satisfied for two economies in both of which the marginal aggregate propensity to consume plus the marginal propensity to invest is greater than unity. (3) One of the two countries *may* be unstable when isolated, provided marginal propensities to consume and invest of the other are sufficiently low. In other words, a country which would be unstable when left to itself may be perfectly stable in a two-economy world because of the dampening influence of low propensities in the other country.

A summary of the three cases examined above may facilitate further discussion. Briefly, I have considered the equilibrium of an economy in isolation, of a single economy dealing with a world whose level of economic activity was regarded as independent of the particular economy studied, and of two economies trading with each other, with due allowances for the reactions of each to changes within the other. The isolated economy was found to be stable if the marginal aggregate propensity to consume plus the marginal propensity to invest were less than unity. Stability of the single economy dealing with a world without reactions was shown to depend upon the sum of the marginal propensity to consume domestic goods and the marginal domestic propensity to invest being less than unity. Stability of the two-economy case is not so easily summarized. In general, one of the two must fulfill the conditions of stability for an isolated economy, both must fulfill the conditions of stability for a single economy dealing with the rest of the world, and the one which does not fulfill the conditions for stability of an isolated economy must have marginal propensities to consume and invest which are sufficiently small to satisfy inequality (8).

The stability conditions developed above may now be used to determine some of the properties of the static system summarized in equations (1). Suppose that shifts occur in the functional relations (1). The equilibrium values u_o, v_o, u_o', and v_o' of the old system will not in general satisfy the new equations. A time sequence of changes, depicted by (2), will ensue and eventually, if the system is stable, a new set of equilibrium values will be approached. It is my purpose to compare the new equilibrium with the one which prevailed before changes in (1) occurred.

Equations (1) are symmetric. Consequently, it will suffice to consider the effects upon both countries of changes in either country alone. I shall assume that the changes occur in Y. Four types of variation will be analyzed. The parameter α of (1) may be interpreted alternatively as an increase in the marginal efficiency of capital or as an increase in public investment. Changes in the average propensity to consume do-

mestic goods are represented by β, while γ is used to indicate a shift in consumption from domestic to foreign goods. Changes in the average propensity to consume foreign goods, accompanied by no change in the average propensity to consume domestic goods, are measured by μ. All movements in consumption and investment functions are uniform shifts, so that marginal propensities remain unchanged.

To analyze the effects of an increased marginal efficiency of capital[11] or an increase in public spending, equations (1) may be differentiated with respect to α, and the resulting simultaneous equations solved for $\partial u/\partial\alpha$, $\partial v/\partial\alpha$, $\partial u'/\partial\alpha$, and $\partial v'/\partial\alpha$, as follows:

$$(9)\qquad \begin{aligned} \frac{\partial u}{\partial \alpha} &= \frac{u_y(1 - v'_{y'} - u'_{1y'})}{\Delta}, & \frac{\partial u'}{\partial \alpha} &= \frac{u'_{y'}u_{2y}}{\Delta}, \\ \frac{\partial v}{\partial \alpha} &= \frac{(1 - u_y)(1 - v'_{y'} - u'_{1y'})}{\Delta}, & \frac{dv'}{\partial \alpha} &= \frac{(1 - u'_{1y'})u_{2y}}{\Delta}, \\ \frac{\partial y}{\partial d} &= \frac{1 - v'_{y'} - u'_{1y'}}{\Delta}, & \frac{\partial y'}{\partial \alpha} &= \frac{u_{2y}}{\Delta}, \end{aligned}$$

where $\Delta \equiv (u_{1y}+v_y-1)(u'_{1y'}+v'_{y'}-1)-u_{2y}u'_{2y'}$ and where u_y is the marginal aggregate propensity to consume in Y (i.e., $u_y \equiv u_{1y}+u_{2y}$), with a similar interpretation of $u'_{y'}$.

No general statement about the sign of Δ may be made. Indeed, it may even be zero, in which case the derivatives (9) do not exist. But if one may assume stability in the trade relations between the two countries—and an explanation of why a slight change does not increase or decrease the values of the variables at exponential rates is otherwise difficult—then the sign of Δ may be determined from our stability conditions. By means of a few transformations, the first of the inequalities of (7) may be reduced to $\Delta>0$. In other words, stability of our two-economy world implies that the denominator of the expressions in (9) is positive. Evaluation of the numerators depends partially upon whether the economies are stable or unstable when isolated. Since both $u_{1y}+v_y$ and $u'_{1y'}+v'_{y'}$ must be less than unity, the only ambiguities are in the values of u_y and $u'_{y'}$, one of which *may* be greater than unity if the economy to which it refers is unstable in isolation.

Thus the following results are obtained: (1) An increase in the marginal efficiency of capital or public spending in Y will increase consumption and income in both countries provided only that the most general

[11] Mr. Keynes devotes a section of his *Treatise on Money* to this problem (Vol. I, Chapter 12, part ii). The adjustments which he envisages are largely price changes, however, since he assumes a constant rate of utilization of the factors of production.

stability conditions (7) are satisfied. (2) If the two countries would have been stable in isolation, then the increased marginal efficiency of capital or spending will increase investment in both countries. Otherwise, the direction of change of investment is indeterminate. From (8), however, it is easily shown that both u_y and $u'_{y'}$ cannot exceed unity. Hence total investment *must* increase in one of the two countries, and it *may* increase in both.

There are striking similarities between the effects of an increase in the marginal efficiency of capital and the effects of an increased average propensity to consume domestic goods, accompanied by no change in the propensity to consume foreign goods. Again differentiating (1), this time with respect to β, and solving for the unknown derivatives, we find:

$$
\begin{aligned}
&\frac{\partial u}{\partial \beta} = 1 + \frac{u_y(1 - v'_{y'} - u'_{1y'})}{\Delta}, && \frac{\partial u'}{\partial \beta} = \frac{u'_{y'}u_{2y}}{\Delta},\\
&\frac{\partial v}{\partial \beta} = -1 + \frac{(1 - u_y)(1 - v'_{y'} - u'_{1y'})}{\Delta}, && \frac{\partial v'}{\partial \beta} = \frac{(1 - u'_{y'})u_{2y}}{\Delta},\\
&\frac{\partial y}{\partial \beta} = \frac{1 - v'_{y'} - u'_{1y'}}{\Delta}, && \frac{\partial y'}{\partial \beta} = \frac{u_{2y}}{\Delta}.
\end{aligned}
\tag{10}
$$

Comparison of (9) with (10) shows that an increase in the average propensity to consume domestic goods in Y affects income, consumption, and investment in Y' in exactly the same way that an increase in the marginal efficiency of capital in the former affects the situation in the latter. In the most general case, income and consumption in Y' will rise with the direction of change of investment remaining indeterminate. But if Y' is a country which would have been stable in isolation, then investment there will also increase. Again the results agree with what we might have expected intuitively. Increased propensity to consume domestic goods in Y means a higher level of income with a consequent rise of imports. And it is by means of these added imports that the income stimulus is transferred from one country to the other, for goods imported by Y are exported by Y'.

Investment in the country with altered consuming habits may either increase or decrease, but if it decreases (because of an unfavorable change in the trade balance) it cannot decrease by as much as consumption increases so that income must rise. The total change in such investment comprises two parts, the change of domestic investment and the change in the trade balance. Increased consumption of domestic goods induces a rise of domestic investment (v_y is positive) so that the first item is positive, but the trade balance may move in either

direction. If it moves *against* the country with altered tastes (i.e., Y), it may counteract the influence of higher domestic investment. To discuss this possibility, the second of equations (10) has been written as follows:

$$\frac{\partial v}{\partial \beta} = \frac{(1 - v'_{y'} - u'_{1y'})(v_y - u_{2y}) + u_{2y}u'_{2y'}}{\Delta}.$$

Thus if v_y exceeds u_{2y}, $\partial v/\partial\beta$ is clearly positive, regardless of special stability conditions. In words, if the marginal propensity to invest domestically exceeds the marginal propensity to consume foreign goods, unfavorable changes in Y's trade balance arising from a higher propensity to consume domestic goods cannot be as large as induced domestic investment. Hence total investment must rise. By another simple transformation of the second of equations (10) it may be shown that $\partial v/\partial\beta$ is always positive if the passive country (Y') is unstable in isolation, regardless of the relation between marginal propensity to invest and marginal propensity to import in the country with altered tastes. This arises, as we shall see presently, from the fact that, with Y' unstable in isolation, higher demand in Y for her own goods moves the trade balance in favor of the latter country. Finally, if Y is the unstable country, and her marginal aggregate propensity to consume is greater than unity, it is clear from (10) that an increase in her propensity to consume domestic goods will reduce her total investment; in this case, the unfavorable change in the trade balance necessarily exceeds the rise of domestic investment.

The third parameter, γ, of (1) represents a shift in consumption in country Y from domestic goods to foreign-made goods. The influence of this shift may be examined by differentiating (1) partially with respect to γ and solving once more for the unknown derivatives.

$$(11)\quad \begin{aligned} \frac{\partial u}{\partial \gamma} &= \frac{-u_y(1 - v'_{y'} - u'_{y'})}{\Delta}, & \frac{\partial u'}{\partial \gamma} &= \frac{u'_{y'}(1 - v_y - u_y)}{\Delta}, \\ \frac{\partial v}{\partial \gamma} &= \frac{-(1 - u_y)(1 - v'_{y'} - u'_{y'})}{\Delta}, & \frac{\partial v'}{\partial \gamma} &= \frac{(1 - u'_{y'})(1 - v_y - u_y)}{\Delta}, \\ \frac{\partial y}{\partial \gamma} &= \frac{-(1 - v'_{y'} - u'_{y'})}{\Delta}, & \frac{\partial y'}{\partial \gamma} &= \frac{1 - v_y - u_y}{\Delta}. \end{aligned}$$

Consider first the income effects. If both countries exhibit stability when isolated, the shift of consumption in Y from domestic to foreign-made goods will lower income there and raise it in Y'. This is the result which might have been expected; diminished foreign investment arising from increased imports reduces the total income of Y, while the addi-

tional exports from Y' serve to increase income of the latter country. But this "normal" case, as I shall call it, is by no means the only possible one. Suppose that Y exhibits stability in isolation while Y' does not. Under such circumstances a shift in Y from domestic to foreign-made goods will *increase* income in both countries. The explanation of this paradoxical result is to be found in the secondary effects of higher income in Y'. Initially, of course, income of the country with changed tastes is reduced as a result of (1) a lower propensity to consume domestic goods and (2) reduced foreign investment. Income of the passive country (Y'), on the other hand, is stimulated by a rise of exports to Y. So far the results do not differ from the case of stability in both countries. If Y' is unstable in isolation, however, there are important repercussions to be considered. In this case her added exports may increase her income considerably—the multiplier effect is large. And eventually, if Y' is unstable in isolation, the secondary movement of goods from the country with changed tastes (Y) to the unstable country (Y') will be sufficient to counteract the depressing influence of the primary shift of consumption in the former country. Hence income rises in both countries.

If Y is an economy which would be unstable in isolation, while Y' is stable, the foregoing conclusion must be reversed; the shift in Y from domestic goods to foreign-made goods will then *reduce* income in both countries. The primary effect, as before, is an increase of income in the country the demand for whose goods has risen (Y') and a reduction of income in the country with altered tastes (Y). Because Y is an unstable country, however, her income movements ultimately dominate the situation. That is, the reduction of her income induces a secondary decline in exports of the passive country, and this decline of exports will then reduce income of the latter country despite the initial stimulus afforded by increased propensity abroad to consume Y' goods. Income therefore declines in both countries.

In explaining the behavior of income in response to a shift of consumption, I have discussed only the secondary changes in the unstable country. Needless to say there are also secondary repercussions in the stable country. Analysis of such repercussions has been omitted not because their quantitative influence is small, but simply because conclusions regarding the final outcome of a particular domestic change always depend upon which country is unstable when isolated.

Both countries cannot be unstable in isolation, for then the two would be unstable also when dealing with each other. Hence a shift of consumption in Y from domestic goods to foreign goods must have one of three results: (1) If both Y and Y' are stable in isolation (the normal case), income will diminish in Y and increase in Y'. (2) If Y is stable

in isolation but Y' is unstable, income will rise in both countries. (3) If Y is unstable in isolation and Y' is stable, income will fall in both countries. The shift in consumption to Y' goods cannot conceivably reduce income in Y' while increasing it in Y, since the conditions necessary to bring about this result contradict our stability hypothesis.

The influence of γ on total consumption and investment in the two countries may now be indicated briefly. Total consumption always moves in the same direction as income. Investment, on the other hand, moves directly or inversely with the movement of income in each country according as the marginal aggregate propensity to consume is less than or greater than unity. Since we know that the marginal aggregate propensity to consume cannot be greater than unity in both countries, it follows that investment *must* move in the same direction as income in either Y or Y' and *may* do so in both.

The fourth parameter to be considered (μ) represents an increase in consumption of foreign goods by Y, with no change in consumption of domestic goods. By the method outlined above it may be shown that

$$
\begin{aligned}
&\frac{\partial u}{\partial \mu} = 1 + \frac{u_y u'_{2y'}}{\Delta}, && \frac{\partial u'}{\partial \mu} = \frac{u'_{y'}(1 - v_y - u_{1y})}{\Delta},\\
&\frac{\partial v}{\partial \mu} = -1 + \frac{(1 - u_y)u'_{2y'}}{\Delta}, && \frac{\partial v'}{\partial \mu} = \frac{(1 - u'_{y'})(1 - v_y - u_{1y})}{\Delta},\\
&\frac{\partial y}{\partial \mu} = \frac{u'_{2y'}}{\Delta}, && \frac{\partial y'}{\partial \mu} = \frac{1 - v_y - u_{1y}}{\Delta}.
\end{aligned}
\tag{12}
$$

If the general stability postulates (7) are accepted, an increase in the average propensity of Y to consume Y' goods must increase total consumption and income in both countries. In the case of Y' this is obvious since the increased demand for Y' goods abroad has resulted in greater foreign investments. In the case of Y it means simply that the stimulating effect of a higher aggregate propensity to consume plus the secondary repercussions of higher income in Y' have exerted a more important influence on income of the former than the depressing consequence of increased imports. Once again, the change of investment in the two countries is indeterminate. If the marginal aggregate propensity to consume is less than unity in Y', investment there must increase; otherwise it will decline. Investment in Y will decline if the marginal aggregate propensity to consume is greater than unity there; otherwise the direction of change is difficult to determine. The second of equations (12), however, may be written as follows:

$$\frac{\partial v}{\partial \mu} = \frac{1}{\Delta}\left[v_y(1 - u'_{1y'} - v'_{y'}) - (1 - u_{1y})(1 - u'_{y'} - v'_{y'})\right]$$

This shows that investment must rise in the country with altered tastes (Y) whenever the passive country (Y') is unstable in isolation. The conclusion, as shown below, depends upon the fact that a higher propensity to consume foreign goods moves the balance of trade in favor of the country with altered tastes whenever the country with which it is dealing is unstable in isolation. Thus if Y' is unstable in isolation both the foreign and domestic components of Y's investment change are positive.

The fundamental equations (1) represent an equilibrium system only in a restricted sense. Except in the special case in which the desire to lend abroad on the part of Y (assuming the trade balance to be favorable) plus the desire of Y' to repatriate funds is exactly equal to the trade balance, the equilibrium of (1) requires annual changes in the indebtedness of one banking system to the other. It is therefore important to know how the changes discussed above will affect the balance of trade. Let $b \equiv u'_1(y') - u_2(y) - \gamma - \mu$ represent this balance.[12] From equations (9) through (12) it is then possible to determine how b changes with movements of $\alpha, \beta, \gamma, \mu$.

$$(13)\qquad \begin{aligned} \frac{\partial b}{\partial \alpha} &= \frac{\partial b}{\partial \beta} = \frac{-u_{2y}(1 - v'_{y'} - u'_{y'})}{\Delta}, \\ \frac{\partial b}{\partial \gamma} &= \frac{-(1 - v_y - u_y)(1 - v'_{y'} - u'_{y'})}{\Delta}, \\ \frac{\partial b}{\partial \mu} &= \frac{-(1 - v_y - u_{1y})(1 - v'_{y'} - u'_{y'})}{\Delta}. \end{aligned}$$

Public spending and increases in the average propensity to consume domestic goods affect the balance of trade in exactly the same way, as might have been expected, since these changes were found to affect incomes in the same way. If Y' is a country which would be stable in isolation, public spending and a higher average propensity to consume domestic goods in Y will move the balance of trade in a direction unfavorable to the expanding country and, *ceteris paribus*, will reduce the balance of Y banks abroad. But if, on the other hand, Y' would be unstable when isolated, then the balance of trade will move in favor of Y. The sequence of events in this case is roughly as follows: (1) imports of the expanding country increase because of greater income arising

[12] It is perhaps needless to say that b represents the trade balance *from the point of view of the country with altered tastes* (i.e., Y).

either from domestic investment or from an increased propensity to consume domestic goods; (2) higher exports of the passive country (Y' in the present instance) exert a stimulating influence on that country's income, with a consequent increase in her demand for goods of the country initiating the change, and (3) if Y' is unstable in isolation, the induced rise of her imports will exceed the rise of her exports occasioned by the initial change in Y. In other words, domestic expansion will lead to a drain on foreign balances only if the country with which the expanding country deals is stable in isolation.

We have seen above that a shift in the propensity to consume from domestic to foreign goods (γ) may (1) decrease income in the country whose tastes have changed, while increasing income of the other country, or (2) increase income in both countries, or (3) reduce income in both. The three possibilities correspond respectively to the three situations in which (1) both countries are stable in isolation, (2) the passive country is unstable in isolation, and (3) the country whose tastes have changed is unstable in isolation. These conclusions, however, afford no clue to the probable change in the balance of trade. But from (13) it is clear that if both countries would be stable in isolation, the shift of consumption must move the trade balance against the country with altered tastes; imports of Y, in other words, must be higher than in the previous equilibrium despite the decline of her income. It is also clear from (13) that if either country would have been *unstable* in isolation, then the shift of consumption will move the balance in *favor* of the country with altered tastes. We may conclude that a decline in the propensity to consume domestic goods accompanied by a rise in the propensity to consume foreign goods will reduce foreign balances of the country with changed tastes only if both countries would be stable when isolated.

How is the situation altered if the rise in demand for foreign goods is accompanied by no decline in demand for domestic goods? The last equation of (13) shows that such a change (μ) causes the balance of trade to move in the same direction (though not necessarily by the same amount) as a similar increase of domestic investment (α). Hence this case need not be discussed further.

All results obtained above are summarized in Tables 1 and 2 where the directions of change of each of the variables of the system with respect to the parameters α, β, γ, and μ are indicated. Table 1-A shows that, without a knowledge of stability conditions, it is difficult to predict the international repercussions of a particular domestic change. Hence in discussing such questions as the foreign-trade aspects of business cycles, effects of capital transfers, and the like, one should always specify which of the countries is assumed to be unstable in isolation

and which stable. Many economists will probably prefer to assume that all countries are stable in isolation, in which case the results given in Table 1-B are relevant.

With stability conditions specified, directions of change may be determined for all variables except investment in one of the two countries. This remaining ambiguity is attributable to the dual nature of such investment. That is, whenever the balance of trade of a country declines

TABLE 1

EQUILIBRIUM MOVEMENTS OF THE SYSTEM FOR THE "GENERAL" AND "NORMAL" CASES

	A. General stability conditions (7) satisfied							B. "Normal" case: both countries stable in isolation						
	u	v	y	u'	v'	y'	b	u	v	y	u'	v'	y'	b
α	+	?	+	+	?	+	?	+	+	+	+	+	+	−
β	+	?	+	+	?	+	?	+	?	+	+	+	+	−
γ	?	?	?	?	?	?	?	−	−	−	+	+	+	−
μ	+	?	+	+	?	+	?	+	?	+	+	+	+	−

TABLE 2

EQUILIBRIUM MOVEMENTS OF THE SYSTEM WITH ONE COUNTRY UNSTABLE IN ISOLATION

	A. Country with altered tastes (Y) unstable in isolation							B. Passive county (Y') unstable in isolation						
	u	v	y	u'	v'	y'	b	u	v	y	u'	v'	y'	b
α	+	?	+	+	+	+	−	+	+	+	+	?	+	+
β	+	?	+	+	+	+	−	+	+	+	+	?	+	+
γ	−	?	−	−	−	−	+	+	+	+	+	?	+	+
μ	+	?	+	+	+	+	−	+	+	+	+	?	+	+

while domestic investment rises (or conversely), the direction of change of total investment cannot be determined without further information. Thus it will be found, upon examination of Tables 1 and 2, that the only doubt which remains when stability conditions are fully specified is in the total investment change of the country whose balance of trade moves in a direction opposite to its income movement. But this is simply the case in which domestic and foreign investment move in opposite

directions, for domestic investment is positively correlated with income. The final outcome under these circumstances depends, as noted above, upon such nonstability conditions as the relation between marginal propensity to invest and marginal propensity to import. In the special case in which the marginal propensity to invest domestically is zero, uncertainty regarding the direction of change of total investment is removed, since changes of foreign investment are then the only *economically determined* changes of total investment.[13] In other words, the induced change of investment for a particular country in a new position of equilibrium is then identical with the change of its trade balance. It follows that, with marginal propensities to invest equal to zero in both countries, changes in consuming habits can only increase investment in one country at the expense of investment in the other.

Apart from investment changes, other aspects of the tables are largely self-explanatory. Hence further comment seems superfluous.

In closing, it may be noted that the use of certain equivalences affords a considerable simplification, provided one is interested in income changes alone. For an isolated community, a given increase in the propensity to consume is known to affect income in exactly the same manner as a corresponding amount of net investment.[14] Similar propositions may be developed for interdependent economies. The fact that $y \equiv u+v$ and $y' \equiv u'+v'$ enables us to write equations (1) as follows:

$$(14) \qquad \begin{aligned} y &= u_1(y) + v(y) + u_2'(y') + \alpha + \beta - \gamma, \\ y' &= u_1'(y') + v'(y') + u_2(y) + \gamma + \mu. \end{aligned}$$

From (14) it is easily shown that: (1) An increase in the propensity to consume domestic goods (β) is equivalent, in its income effect, to a corresponding amount of net domestic investment (α). (2) A shift in the propensity to consume from domestic to foreign goods (γ) is equivalent to net disinvestment of γ in the country whose tastes have changed, accompanied by net investment of the same amount in the second country. (3) An increase in the average propensity of one of the countries to consume foreign goods (μ) is equivalent to a corresponding amount of net investment in the other country. Thus all of the changes which I have considered may be reduced, for income analysis, to combinations of net investment changes in the two countries.

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[13] This is the system envisaged by those who object to the inclusion of induced investment as a component of static equilibrium.

[14] Cf. P. A. Samuelson, "Theory of Pump-Priming Reëxamined," *American Economic Review*, Vol. 30, No. 3, September, 1940, p. 505.